II. *De Inventione Centri Oscillationis. Per* Brook Taylor *Armig. Regal. Societat. Sodal.*

Definitio.

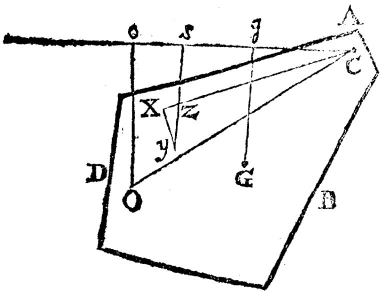
Est Centrum Oscillationis punctum quoddam in corpore pendulo, cujus vibrationes singulæ eodem modo atq; eodem tempore peraguntur, ac si illud solum ad eandem distantiam a puncto suspensionis filo suspenderetur.

PER se vix satis manifestum est in corpore aliquo dari hujusmodi punctum: utpote cujus acceleratio debeat, (*per hanc def.*) in omnibus inclinationibus corporis penduli ad Horizontem, perinde esse, ac si a propriâ tantùm gravitate urgeatur; reliquis particulis totius corporis ejus motum proprium haud perturbantibus. Itaq; in ordine ad inventionem hujus Centri, præmittenda est una atq; altera propositio, unde constet tale punctum dari.

Prop. I. Prob. I

In corporis Oscillantis datâ quâvis inclinatione ad Horizontem, invenire punctum cujus acceleratio perinde sit, ac si ab ipsius propriâ tantum gravitate urgeatur.

Sit A B D corporis propositi sectio in plano ad Horizontem perpendiculari, in quo movetur centrum gravitatis G, centro suspensionis existente C. Distinguatur corpus in elementa prismatica plano A B D perpendicularia,



cularia, adeoque Horizonti semper parallela; ut facile patebit ex motu centri gravitatis G in plano illo A B D. Atq; ob hujusmodi situm, tale elementum quodvis spectari potest tanquam punctum Physicum p in plano eodem A B D ad punctum z locatum. Reducatur itaq; corpus propositum in planum Physicum A B D constans ex hujusmodi particulis p.

In hoc plano ut inveniatur punctum O, cujus acceleratio propria non mutatur ab actionibus particularum reliquarum, attendendum est ad vires particulæ cujusvis singularis p in puncto z sitæ. Nam ex hisce viribus conjunctis oritur plani totius motus absolutus; cujus ope datur motus puncti cujusvis propositi; unde vicissim invenitur punctum cujus motus est datus.

At urgetur particula p a vi propriæ gravitatis; quæ si partium cohæsio dissolveretur, in dato tempore minimo, datam produceret accelerationem motûs in perpendiculari ad Horizontem zy. Ad Cz duc normalem yx, & resolvetur acceleratio zy in partes zx & xy. Ob corporis rigiditatem, tollitur vis zx per resistentiam puncti C. At vi reliquâ xy trahitur spatium ABD in gyrum circa punctum C; & ductâ horizontali Co & perpendiculari zs, erit ea ut $\frac{Cs}{Cz}$: Nempe ob gravitatis vim datam, & similia triangula xyz & sCz. Ergo vis particulæ p ad movendum spatium ABD est ut $\frac{Cs}{Cz} \times p$.

Ad has vires in unum colligendas, sit O punctum invariabile, in lineâ ad libitum ducta & ad distantiam adhuc incognitam CO. Tum erit vis particulæ p ad movendum

vendum punctum O, ut $\frac{Cz}{CO} \times \frac{Cs}{Cz} \times p$, hoc est ut $\frac{Cs}{CO} \times p$. Acceleratio autem, quam tribuit p eidem puncto O, erit ut $\frac{CO}{Cz} \times \frac{Cs}{Cz}$. Itaq; applicatâ vi illâ $\frac{Cs}{CO} \times p$ ad hanc accelerationem $\frac{CO \times Cs}{Czq:}$, erit quotiens $\frac{Czq:}{COq:} \times p$ particula, quæ, si in ipso puncto O fingatur moveri cum eâdem acceleratione $\frac{CO \times Cs}{Czq:}$, eundem omnino produceret motum, quem in eodem puncto O producit particula p. Hinc demum reducitur Problema ad motuum Theorema notissimum: Applicatâ enim summâ virium $\frac{Cs}{CO} \times p$ ad summam particularum $\frac{Czq:}{COq:} \times p$, erit quotiens acceleratio absoluta puncti O. Dein ductâ perpendiculari Oo, & positâ hac acceleratione æquali datæ accelerationi $\frac{Co}{CO}$ ipsius puncti O, dabitur distantia CO. Sit enim $\frac{Co}{CO} = d$, & (juxta methodum *Fluxionum*) $Cs \times p = \dot{M}$, & $Czq: \times p = \dot{C}$. Tum ob CO invariabilem erit summa omnium virium $\frac{Cs}{CO} \times p = \frac{M}{CO}$, & summa omnium particularum $\frac{Czq:}{COq:} \times p = \frac{C}{COq:}$. Unde, applicatâ summâ momentorum ad summam corporum, erit $\frac{M}{C} \times CO = d$ adeoq; $CO = \frac{dC}{M}$. Inventis igitur C & M, per *Fluxionum* methodum inversam, dabitur CO. Q. E. I.

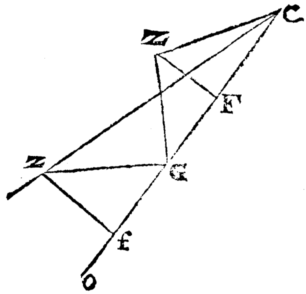
Cor. A centro gravitatis G ad horizontalem Co duc perpendicularem Gg, & sit corpus ipsum ABC = A. Tum

Tum ex notiſſimâ indole centri gravitatis erit $M = Cg \times A$. Unde eſt $CO = \frac{dC}{Cg \times A}$.

Prop. 2. Theor. 1.

Iiſdem poſitis, quæratur punctum O in rectâ C G tranſe-

$= CGq: + Gzq: + 2CG \times Gf$. Eſt ergo $C =$ (aggregato omnium $Czq: \times p =$) aggregato omnium $CGq: \times p + Gzq: \times p - 2CG \times GF \times p + 2CG \times Gf \times p$. At ob centrum gravitatis G, eſt aggregatum omnium $2CG \times GF \times p =$ aggregato omnium $2CG \times Gf \times p$. Quare eſt $C =$ aggregato omnium $CGq: \times p + Gzq: \times p = CGq: \times A + D$. At enim per *Theor.* 1. eſt $CO = \frac{C}{CG \times A}$. Ergo $CO = CG + \frac{D}{CG \times A}$. Q. E. D.



Cor. Hinc datur parallelogrammum $CG \times GO$. Eſt enim $GO = \frac{D}{CG \times A}$. At dantur A & D. Quare datur $CG \times GO = \frac{D}{A}$.

Prop. 4. Theor. 3.

Iiſdem poſitis, ſi in puncto O conſtituatur particula phyſica $\frac{CG \times A}{CO}$, quæ propria gravitate agitata Oſcillet circa punctum C; ſpatij A B C motus perinde omnino erit, ac ſi agitaretur ab Oſcillatione ipſius corporis A.

Conſtat tam ex Natura centri gravitatis, quam *per Prob* 1. Eſt enim $\frac{CG \times A}{CO}$ aggregatum omnium $\frac{Czq: \times p}{COq:} = \frac{C}{COq:}$.

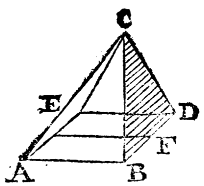
Prop. 5. Prob. 2.

Datis corporis cujusvis magnitudine A, centro gravitatis G, & puncto suspensionis C. Invenire ejusdem centrum Oscillationis O.

Fit per Theor 1. inveniendo quantitatem C; vel per Theor. 2. quærendo quantitatem D.

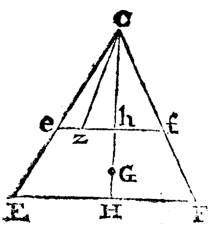
Scholium.

Ad instituendum calculum in casu particulari, eligenda est quantitas C vel D, prout suggerit natura figuræ propositæ. Dein datâ earum alterutrâ, altera item dabitur per æquationem (*Prop.* 3.) $C = CGq : \times A + D$. Unde etiam dabitur pgr. $CG \times GO = \frac{D}{A}$ (*Cor. Prop.* 3.) $= \frac{C}{A} - CGq :$. Cujus ope, ex datis centro gravitatis & puncto suspensionis, datur centrum Oscillationis per solam divisionem. Quare in quolibet exemplo semper commodissimum erit hoc parellelogrammum primùm eruere, vel per computum ipsius D, vel per quantitatem C, ex idoneâ assumptione centri suspensionis.



Superest, ut hæc exemplis aliquot illustremus.

Ex. 1. Sit figura proposita Pyramis A D C, cujus basis est pgr. A D, sitque motus centri gravitatis in plano transeunte per verticem C & diametrum basis E F lateri A B parallelam.



Ad calculum commodissime instituendum, sit ipse vertex C centrum suspensionis. Tum ad modum *Prob.* 1. reducatur figura ad planum physicum trianguli Isoscelis C E F, in quo e f parallela ipsi EF repræsentat lineam physicam ex particulis p compositam. Sit C H = a.

H F

$HF = b$, & $Ch = x$. Tum ex naturâ figuræ erit $eh = \frac{bx}{a}$, & particula p sita ad punctum z erit ut x; vel Potius, facto $hz = v$, erit $\dot{v}\,\dot{x}$ elementi prismatici basis, & p erit ut $\dot{v}\,\dot{x}\,x$. Unde erit $\dot{C} = Czq: \times \dot{v}\,\dot{x}\,x = \dot{v}\,x\,x^3 + \dot{x}\,\dot{v}\,v^2 x$. Ideoq; summa omnium $Czq: \times p$ in lineâ hz erit $v\,\dot{x}\,x^3 + \frac{\dot{x}\,x\,v^3}{3}$; & in lineâ ef (pro v ponendo $\frac{bx}{a}$) erit summa illa $\frac{6ba^2 + 2b^3}{3a^3} \times \dot{x}\,x^4$. Unde iterum capiendo fluentem, & pro x scribendo a, erit $C = \frac{6ba^2 + 2b^3}{15} \times a^2$. Est autem pyramis ipsa $A = \frac{2baa}{3}$, & distantia centri gravitatis G a vertice C est $CG = \frac{3}{4}a$. Unde $\frac{C}{A} - CGq: = \frac{D}{A} = CG \times GO = \frac{3a^2 + 16b^2}{80}$.

Ex. 2. Sit figura proposita Conus rectus descriptus rotatione trianguli isoscelis ECF circa perpendiculum CH.

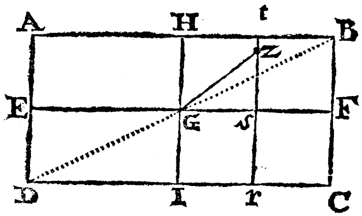
Hic iterum sumpto vertice C pro centro suspensionis, & factis $CH = a$, $HE = b$, $Ch = x$, $hz = v$, ut supra; erit $p = 2\,\dot{x}\,\dot{v} \times \sqrt{\frac{bb}{aa}xx - vv}$; unde $\dot{C} = 2\,\dot{v}\,\dot{x}\,\overline{\times\, xx + vv} \times \sqrt{\frac{bb}{aa}xx - vv}$. Sit B segmentum circuli diametro ef descripti, quod adjacet Abscissæ $hz = v$,

 & Or.

& Ordinatæ $\sqrt{\frac{bb}{aa}xx - vv}$; tum erit summa omnium $Czq : \times p$ in rectâ $hz = 2\dot{x} \times \frac{4a^2 + b^2}{4a^2} x^2 B - \frac{1}{3}\dot{x}v \times \overline{\frac{b^2}{a^2}x^2 - v^2}\Big|^{\frac{3}{2}}$. Et quando $v = eb$, erit hæc summa $2\dot{x} \times \frac{4a^2 + b^2}{4a^2} x^2 B$; cujus duplum $\frac{4a^2 + b^2}{a^2}\dot{x}x^2 B$ est pars ipsius C in rectâ ef. Est autem area B ut x^2; sit ergo $B = cx^2$; atq; pars illa ipsius C erit $\frac{4a^2 + b^2}{a^2} \times c\dot{x}x^4$. Unde capiendo fluentem erit $C = \frac{4a^2 + b^2}{5} \times ca^3$. Est autem conus ipse $A = \frac{4}{3}ca^3$, & $CG = \frac{3}{4}a$. Unde $\frac{C}{A} - CGq := \frac{D}{A} = \frac{3a^2 + 12b^2}{80}$.

Atq; ad hunc modum procedit calculus in alijs figuris, ubi rationes Ch ad he, & hz ad p sunt magis compositæ.

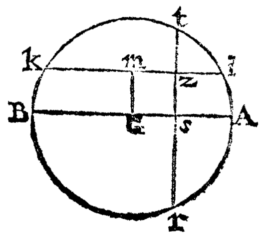
Ex. 3. Ut pateat ratio calculi quantitatis D, sit figura proposita parallelepipedon, cujus facies Horizonti perpendicularis, & parallela plano motûs centri gravitatis est ABD. Duc diametros EF & HI, & sit altitudo elementorum p, :: & sit tr parallela HI; & $GF = a$, $GH = b$, $Gs = x$, & $sz = v$. Tum erit $\dot{D} = \dot{v}\dot{x}xx + \dot{x}\dot{v}vv$. Unde ipsius D pars in rectâ tr erit $2b\dot{x}x^2 + \frac{2}{3}b^3\dot{x}$: atq; iterum sumendo fluentis duplum, erit



D

$D = \frac{4ba^3 + 4b^3a}{3}$. Atqui est $A = 4ab$; unde est $\frac{D}{A} = \frac{aa + bb}{3} = \frac{1}{12}$ D B quad.

Ex. 4. Sit ultimum exemplum in Sphærâ, cujus circulus maximus B t r, diameter A B, & centrum G. Tum ductis lineis ut in Schemate satis patent, erit $\dot{D} = G\,s\,q: \times \dot{p} + Gm\,q: \times \dot{p}$. At summa omnium $G\,s\,q: \times \dot{p}$ in recta t r est $G\,s\,q$: ductum in aream circuli diametro t r descripti. Item summa omnium $G\,M\,q: \times \dot{p}$ in rectâ k i est $G\,m\,q: \times$ aream circuli diametro k i descripti. Unde statim constat esse D = quater fluenti ipsius $G\,s\,q$: in aream circuli cujus diameter est t r. Sit ergo c area circuli cujus radij quadratum est 1, & sit $G\,A = a$, & $G\,s = x$. Tum erit $\dot{D} = 4\dot{x}\,xx \times caa - cxx = 4ca^2\dot{x}x^2 - 4c\dot{x}x^4$. Unde sumendo fluentem, & faciendo $x = a$, erit $D = \frac{8}{15}ca^5$. Est autem $A = \frac{4}{3}ca^3$.



Unde $\frac{D}{A} = \frac{2}{5}aa$.

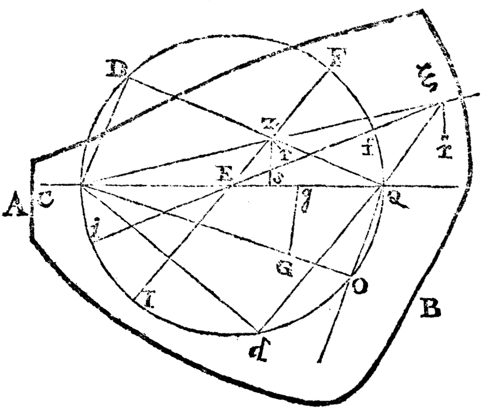
Ob affinitatem solutionis libet his subjungere Problema de inventione Centri Percussionis.

Prop. 6. Prob. 3.

Corporis cujusvis circa datum punctum rotati, invenire Centrum Percussionis; punctum scilicet tale, ut Corpus in illud impingens, & eâdem operâ solutum a puncto suspensionis, neque huc neque illuc inclinet.

Primùm constat hoc punctum quæri debere in plano motùs centri gravitatis. Si enim corpus resolvatur in e-

 lementa



lementa prismatica plano isti normalia, ferentur ea motu sibi parallelo; unde momenta ex utrâq; parte istius plani erunt æqualia; adeoq; per resistentiam factam in hoc plano, corporis punctum nullum de eo pelletur. Sit ergo planum illud A B, ad quod reducetur corpus per contractionem elementorum prismaticorum in particulas p ad puncta z sitas, ut in *Prob.* 1. In hoc plano sit C centrum rotationis; aut saltem ejus projectio facta per lineam perpendicularem in hoc planum demissam; & sit Q punctum quæsitum. Per C duc ad libitum C ξ, in quâ sume puncta duo z & ξ, ita ut ductis z Q & ξ Q, sit angulus C z Q obtusus, & angulus C ξ Q acutus: atque in punctis z & ξ sint particulæ p & π. Tum ad C ξ ductis normalibus z r & ξ *r*, quæ sint ad inuicem ut C z ad C ξ, ijs repræsentabuntur velocitates absolutæ particularum p & π. At harum velocitatum partes quæ sunt in directionibus z Q & ξ Q, tolluntur per resistentiam puncti Q. Ad Q z & Q ξ duc normales C D & C d, & ob angulos æquales z C D = r z Q, & ξ C d = *r* ξ Q, velocitatum partes reliquæ, in directionibus ipsis Q z & Q ξ perpendicularibus, erunt ut z D & ξ d. Unde habitâ ratione distantiarum Q z & Q ξ, erunt vires particularum p & π ad movendum spatium A B in partes contrarias, ut D z × z Q × p, & d ξ × ξ Q × p. At per conditiones Problematis debent summæ hujusmodi contrariarum virium esse inter se æquales.

Ob

Ob angulos ad D & d rectos, sunt puncta D & d ad circumferentiam circuli diametro C Q descripti. Sit istius circuli centrum E. Tum ductis E z & E ξ circulo occurrentibus in F & I, f & i, erit D z $\times$ z Q = F z $\times$ z I = E F q : — E z q : = E Q q : — E z q :, & d ξ $\times$ ξ Q = E ξ q : — E Q q : Quare erit summa omnium E Q q : $\times$ p — E z q : $\times$ p = summæ omnium E ξ q : $\times$ π — E Q q : $\times$ π; & terminis transpositis, summa omnium E Q q : $\times$ $\overline{p + \pi}$: = summæ omnium E z q : $\times$ p + E ξ q : $\times$ π, hoc est, si p ponatur tam pro particulâ p intra circulum, quam pro particula π extra circulum, erit summa omnium E Q q : $\times$ p = summæ omnium E z q : $\times$ p. Ad C Q duc normalem z s. Tum erit E z q : = C z q : + E C q : — Q C $\times$ C s. Quo valore ipsius E z q : ei substituto, & æquatione debité tractatâ, tandem invenies summam omnium C Q $\times$ C s $\times$ p = summæ omnium C z q : $\times$ p. Unde est C Q $= \frac{\text{summæ omnium C z q} : \times \text{p}}{\text{summ: omnium C s} \times \text{p}}$. At enim est summa omnium C z q : $\times$ p ipsa quantitas C in calculo centri Oscillationis : & si centrum gravitatis sit G, & ad C Q ducatur normalis G g, & corpus ipsum dicatur A, erit summa omnium C s $\times$ p = C g $\times$ A. Unde est C Q $= \frac{\text{C}}{\text{Cg} \times \text{A}}$. Sit centrum Oscillationis O; tum per *Theor.* 1. erit C O $= \frac{\text{C}}{\text{CG} \times \text{A}}$. Unde est C g : C G : : C O : C Q. Quare per O ducta ad C O perpendicularis transibit per punctum Q. Q. E. I.

III. *Epistola*